

HW 3 — Due: Sep 11

Lecturer: Prapun Suksompong, Ph.D.

**Instructions**

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
- (b) It is important that you try to solve all problems. (5 pt)  
The extra question at the end is optional.
- (c) Late submission will be heavily penalized.
- (d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.



**Problem 1.**

(a) Suppose that  $P(A|B) = 0.4$  and  $P(B) = 0.5$  Determine the following:

(i)  $P(A \cap B) = 0.2$

(ii)  $P(A^c \cap B) = P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3$

(iii)  $P(B^c \cap A)$

can be anything between 0 to 0.5.

[Montgomery and Runger, 2010, Q2-105]

(b) Suppose that  $P(A|B) = 0.2$ ,  $P(A|B^c) = 0.3$  and  $P(B) = 0.8$  What is  $P(A)$ ? [Montgomery and Runger, 2010, Q2-106]

$P(A \cup B) = ?$

$P(A^c \cup B) = ?$

**Problem 2.** Due to an Internet configuration error, packets sent from New York to Los Angeles are routed through El Paso, Texas with probability  $3/4$ . Given that a packet is routed through El Paso, suppose it has conditional probability  $1/3$  of being dropped. Given that a packet is not routed through El Paso, suppose it has conditional probability  $1/4$  of being dropped.

- (a) Find the probability that a packet is dropped.  
Hint: Use total probability theorem.

- (b) Find the conditional probability that a packet is routed through El Paso given that it is not dropped.

Hint: Use Bayes' theorem.

[Gubner, 2006, Ex.1.20]

**Problem 3.** You have two coins, a fair one with probability of heads  $\frac{1}{2}$  and an unfair one with probability of heads  $\frac{1}{3}$ , but otherwise identical. A coin is selected at random and tossed, falling heads up. How likely is it that it is the fair one? [Capinski and Zastawniak, 2003, Q7.28]

**Problem 4.** You have three coins in your pocket, two fair ones but the third biased with probability of heads  $p$  and tails  $1 - p$ . One coin selected at random drops to the floor, landing heads up. How likely is it that it is one of the fair coins? [Capinski and Zastawniak, 2003, Q7.29]

**Problem 5.** Someone has rolled a fair dice twice. You know that one of the rolls turned up a face value of six. What is the probability that the other roll turned up a six as well? [Tijms, 2007, Example 8.1, p. 244]

Hint: Not  $\frac{1}{6}$ .

$$\begin{aligned}
 P(+|H^c) &= 0.01 \\
 P(-|H^c) &= 0.99 \\
 P(H) &= \frac{1}{5000} \\
 P(+|H) &= 0.99 \\
 P(-|H) &= 0.01
 \end{aligned}$$

**Problem 6.** Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (-) response. Suppose the test gives the correct answer 99% of the time.

- (a) What is  $P(-|H)$ , the conditional probability that a person tests negative given that the person does have the HIV virus?  $P(-|H) = 1 - P(+|H) = 1 - 0.99 = 0.01$

- (b) What is  $P(H|+)$ , the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive?

$$P(H|+) = \frac{P(H \cap +)}{P(+)} = \frac{P(+|H)P(H)}{P(+)}$$

### Extra Question

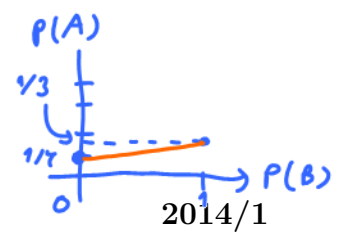
Here are optional questions for those who want more practice.

**Problem 7.**

$$\begin{aligned}
 P(+)&= P(+ \cap H) + P(+ \cap H^c) \\
 &= P(+|H)P(H) + P(+|H^c)P(H^c) \\
 &= 0.99 \cdot \frac{1}{5000} + 0.01 \cdot \left(1 - \frac{1}{5000}\right)
 \end{aligned}$$

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

$$= \frac{1}{3}P(B) + \frac{1}{4}P(B^c)$$



- (a) Suppose that  $P(A|B) = 1/3$  and  $P(A|B^c) = 1/4$ . Find the range of the possible values for  $P(A)$ .
- (b) Suppose that  $C_1, C_2$ , and  $C_3$  partition  $\Omega$ . Furthermore, suppose we know that  $P(A|C_1) = 1/3$ ,  $P(A|C_2) = 1/4$  and  $P(A|C_3) = 1/5$ . Find the range of the possible values for  $P(A)$ .

$$P(A) = \underbrace{P(A|C_1)}_{1/3} P(C_1) + \underbrace{P(A|C_2)}_{1/4} P(C_2) + \underbrace{P(A|C_3)}_{1/5} P(C_3) \quad P(A) \in \left(\frac{1}{3}, \frac{1}{5}\right)$$

**Problem 8.** Software to detect fraud in consumer phone cards tracks the number of metropolitan areas where calls originate each day. It is found that 1% of the legitimate users originate calls from two or more metropolitan areas in a single day. However, 30% of fraudulent users originate calls from two or more metropolitan areas in a single day. The proportion of fraudulent users is 0.01%. If the same user originates calls from two or more metropolitan areas in a single day, what is the probability that the user is fraudulent? [Montgomery and Runger, 2010, Q2-144]

**Problem 9.** In his book *Chances: Risk and Odds in Everyday Life*, James Burke says that there is a 72% chance a polygraph test (lie detector test) will catch a person who is, in fact, lying. Furthermore, there is approximately a 7% chance that the polygraph will falsely accuse someone of lying.

- (a) If the polygraph indicated that 30% of the questions were answered with lies, what would you estimate for the actual percentage of lies in the answers?
- (b) If the polygraph indicated that 70% of the questions were answered with lies, what would you estimate for the actual percentage of lies?

[Brase and Brase, 2011, Q4.2.26]

**Problem 10.** An article in the British Medical Journal [“Comparison of Treatment of Renal Calculi by Operative Surgery, Percutaneous Nephrolithotomy, and Extracorporeal Shock Wave Lithotripsy” (1986, Vol. 82, pp. 879-892)] provided the following discussion of success rates in kidney stone removals. Open surgery (OS) had a success rate of 78% (273/350) while a newer method, percutaneous nephrolithotomy (PN), had a success rate of 83% (289/350). This newer method looked better, but the results changed when stone diameter was considered. For stones with diameters less than two centimeters, 93% (81/87) of cases of open surgery were successful compared with only 87% (234/270) of cases of PN. For stones greater than or equal to two centimeters, the success rates were 73% (192/263) and 69% (55/80) for open surgery and PN, respectively. Open surgery is better for both stone sizes, but less successful in total. In 1951, E. H. Simpson pointed out this apparent contradiction (known as Simpson’s Paradox) but the hazard still persists today. Explain how open surgery can be better for both stone sizes but worse in total. [Montgomery and Runger, 2010, Q2-115]